

Matrix product states for quantum metrology

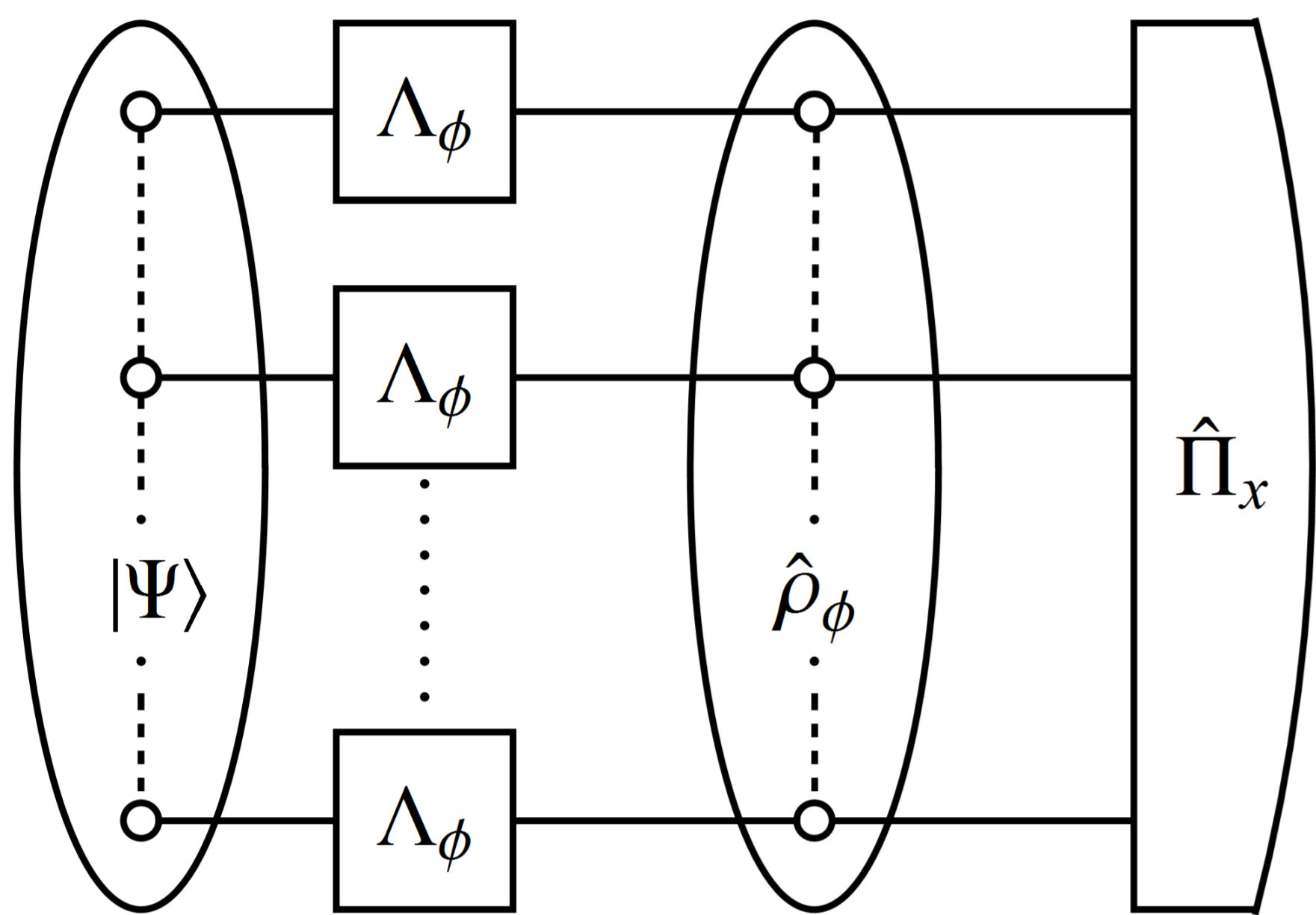
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arXiv:1301.4246

Introduction:

Quantum metrology gives bounds on achievable precision in measurements of parameters like phase delays, frequencies etc. However it is not known if those bounds are saturable and what is the optimal state, i.e. what quantum resources like entanglement should be used to get it. It is known that asymptotically optimal states should be tensor products of some states with lower particle number, thus they would not be highly entangled. Natural class of states which exhibits similar properties are matrix product states and we show that indeed, they are good to solve optimization problems in quantum metrology.

General scheme:



- $\Lambda_\phi = \hat{U}_\phi$
 $\Rightarrow \Delta\phi \approx 1/N$
No decoherence \rightarrow
Heisenberg scaling
- $\Lambda_\phi = \sum_i K_i \bullet K_i^\dagger$
 $\Rightarrow \Delta\phi \approx c/\sqrt{N}$
Decoherence \rightarrow
SQL-like scaling

Traditionally:

Measurement of observable \hat{A} : $\Delta\phi = \Delta\hat{A} / \left| \frac{d\langle\hat{A}\rangle}{d\phi} \right|$

In general – need for optimization over $\hat{\Pi}_x$, $\tilde{\phi}(x)$ and input state $|\psi\rangle$
 \rightarrow very hard task.

Cramer-Rao inequality:

$$\Delta\phi \geq 1/\sqrt{F_\phi}, \quad F_\phi = \text{Tr}(\hat{\rho}_\phi \hat{L}_\phi^2) \quad \begin{array}{l} F - \text{quantum Fisher} \\ \text{information (QFI).} \\ L - \text{symmetric} \\ \text{logarithmic derivative.} \end{array}$$

$$\frac{d\hat{\rho}_\phi}{d\phi} = \frac{1}{2}(\hat{L}_\phi \hat{\rho}_\phi + \hat{\rho}_\phi \hat{L}_\phi)$$

C-R inequality gives bound optimized over all $\hat{\Pi}_x$ and $\tilde{\phi}(x)$. State optimization remains.

Matrix product states:

How to describe a state of N, two-level particles?

In general:

$$|\psi\rangle = \sum_{\sigma_1 \dots \sigma_N=0}^1 c_{\sigma_1 \dots \sigma_N} |\sigma_1 \dots \sigma_N\rangle$$

2^N coefficients.

Symmetric subspace:

$$|\psi\rangle = \sum_{n=0}^N c_n |n, N-n\rangle$$

$N+1$ coefficients.

Matrix product state (MPS):

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{\sigma_1 \dots \sigma_N=0}^1 \text{Tr}(A_{\sigma_1} \dots A_{\sigma_N}) |\sigma_1 \dots \sigma_N\rangle$$

$2D^2N$ coefficients of some matrices A .

Symmetric subspace:

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{n=0}^N \sqrt{\binom{N}{n}} \text{Tr}(A_0^n A_1^{N-n}) |n, N-n\rangle$$

$2D$ coefficients.

Advantages:

- Local description
- Any state can be described by MPS, perhaps with large D .
- Very good description of states with local correlations.

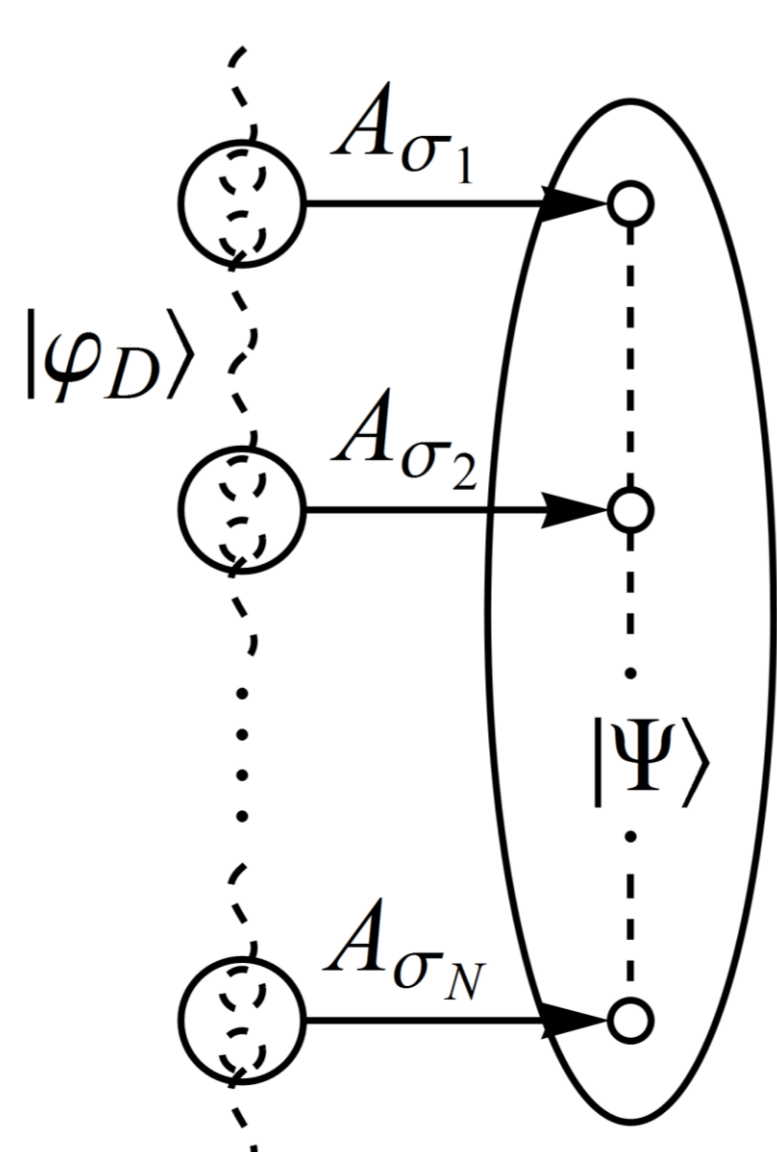
Intuition:

• Assume that each particle is substituted by a pair of D dimensional virtual systems.

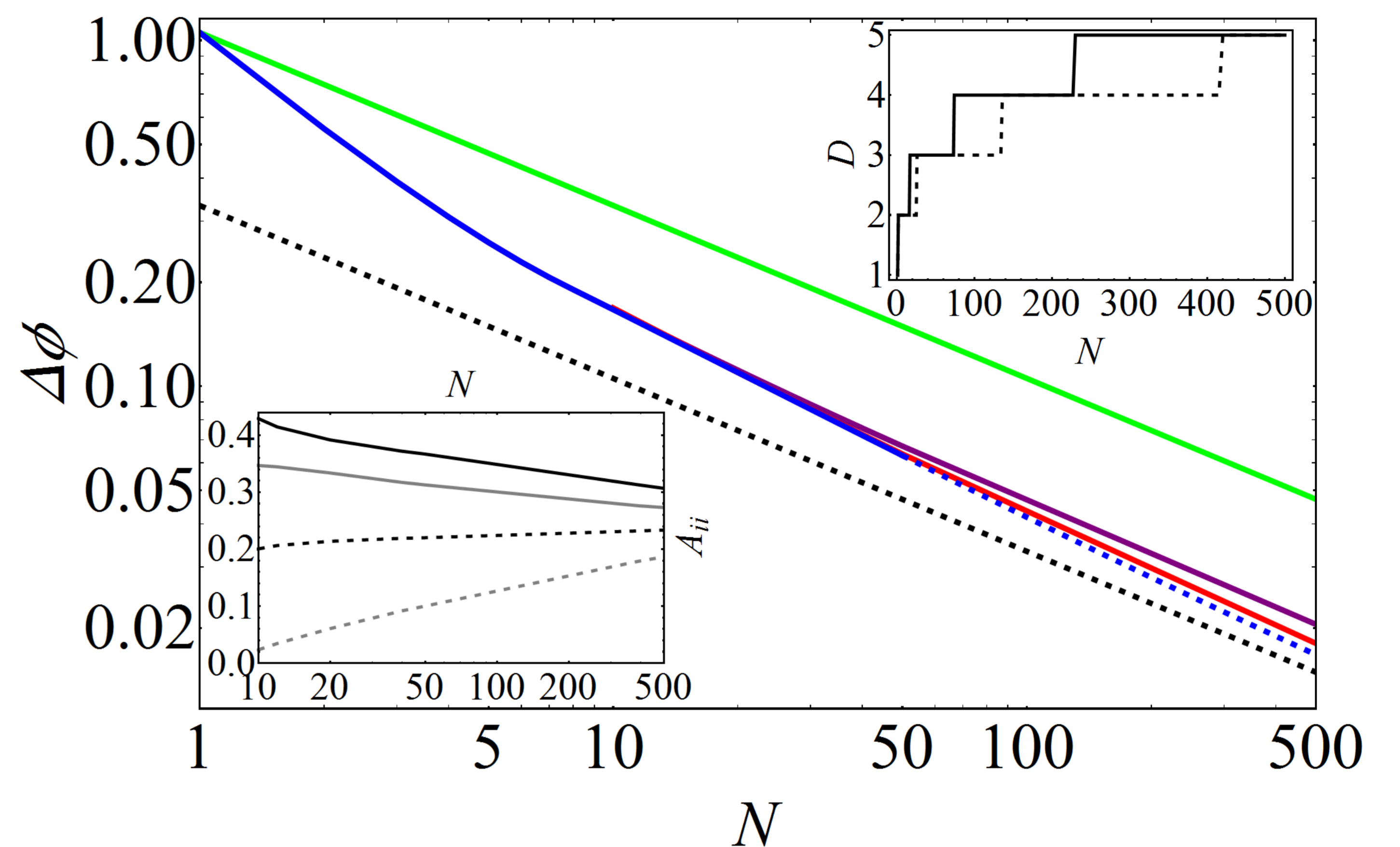
• Neighboring systems are in state

$$|\varphi_D\rangle = \frac{1}{\sqrt{D}} \sum_{j=1}^D |j, j\rangle$$

• Apply maps $A_{\sigma_i} = \sum_{\alpha\beta\sigma_i} A_{\alpha\beta\sigma_i} |\sigma_i\rangle \langle\alpha\beta|$ to each pair of virtual systems.

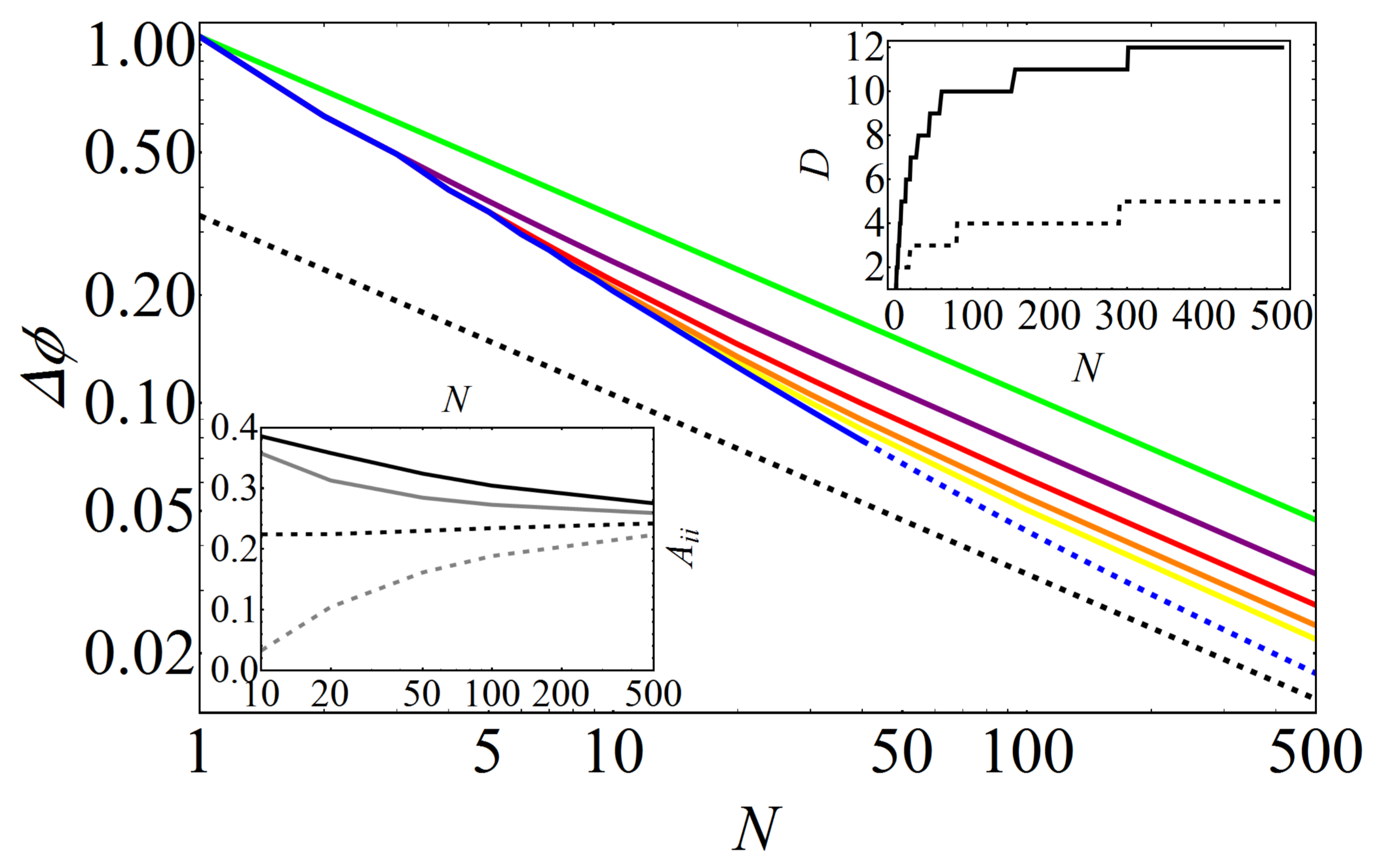


Results: QFI and loss of probes



Cramer-Rao bound optimized over MPS with growing bond dimension D as a function of total photon number N ; $D=1$, green, $D=2$, purple, $D=3$, red. Blue curve is bound obtained after direct optimization and blue dashed curve is its approximation for higher N . Black, dashed line is ultimate asymptotic bound. Upper inset is minimal D required to get optimal precision with at most 1% discrepancy. Lower inset are diagonal elements of optimal matrix as a function of number of particles in the setup.

Results: Ramsey spectroscopy and loss of probes



Precision obtained for Ramsey spectroscopy setup optimized over MPS with growing bond dimension D as a function of total photon number N ; $D=1$, green, $D=2$, purple, $D=3$, red, $D=4$, orange, $D=5$, yellow. Blue curve is result obtained after direct optimization and blue dashed curve is its approximation for higher N . Black, dashed line is ultimate asymptotic bound. Upper inset is minimal D required to get optimal precision with at most 1% discrepancy. Lower inset are diagonal elements of optimal matrix as a function of number of particles in the setup.

Summary:

In summary, MPS are very efficient and suitable to solve optimization problems in quantum metrology. They also gives some insight into nature of optimal states – in the presence of decoherence all diagonal values of MPS matrices are complementarily ordered – highest with the lowest one etc. and they converge to the same value which gives a product state.

Acknowledgments:

We would like to thank to QUASAR and TEAM projects and to Foundation for Polish Science